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Negative parity states of ^{109}Ag in the intermediate-coupling unified nuclear model

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Abstract. Application of the particle–harmonic vibrator intermediate-coupling model to the negative parity states of ^{109}Ag yields good agreement with the experimental level spectrum but has limited success in reproducing E2 and M1 properties. Better overall agreement with experiment is obtained on applying the quasiparticle–anharmonic vibrator version of the model.

1. Introduction

As a result of recent experiments (Robinson *et al* 1970, Thomas *et al* 1973) most of the electromagnetic transition rates and some of the static moments for the lowest five negative parity states in ^{109}Ag are now well known. In a previous preliminary analysis of these properties by Throop *et al* (1972) it was pointed out that a satisfactory description of the electromagnetic properties of the lowest three states (ie $1/2^-$ ground state, $3/2^-$ at 311 keV and $5/2^-$ at 415 keV) is provided by the weak-coupling core-excitation model of de-Shalit (1961). According to this model the low-lying negative parity states in ^{109}Ag may be assumed to arise from the coupling of a $p_{1/2}$ proton hole to the collective core states in the neighbouring ^{110}Cd nucleus.

This simple model breaks down, however, when applied to the second $3/2^-$ (702 keV) and $5/2^-$ (863 keV) states which are the likely candidates for members of the model's two-phonon doublet. Thus, although strong E2 transitions from the 863 keV level to the lowest $3/2^-$ and $5/2^-$ levels have been measured (Robinson *et al* 1970), neither the sum nor the ratio of the B(E2) values agrees with the model's predictions. Further, the first and second $3/2^-$ states both display appreciable ground state M1 transitions which are forbidden on this model.

Ford *et al* (1970) and Robinson *et al* (1970) were able to account for the observed E2 transitions, as well as for proton inelastic scattering data involving the second $3/2^-$ and $5/2^-$ levels, on the basis of the phonon-mixing model of Tamura and Udagawa (1966). It was shown from that analysis that each of the two states contained a large two-phonon and a small one-phonon component but with a total probability for both of less than 30% of the state, the remaining part being unspecified.

It seemed from this that an improved theoretical description might be achieved by including more single-particle and collective degrees of freedom in the calculation. With this motivation we have carried out two intermediate-coupling model (ICM) calculations for the negative parity states in ^{109}Ag. Calculation A, which is presented

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in § 3, is based on the standard particle–harmonic vibrator version of the ICM; the method and formalism used in the calculation are outlined in § 2. Calculation **B**, which is presented in § 4, follows the quasiparticle–anharmonic vibrator version of the ICM, recently suggested and employed by Castel *et al* (1971). The results of each calculation are presented and compared with available experimental data. A summary of conclusions is given in § 5.

2. Formalism and method of calculation

Calculation A is based entirely on the classical intermediate-coupling unified nuclear model (Bohr 1952, Bohr and Mottelson 1953, Choudhury 1954, Ford and Levinson 1955); a brief summary of the model's formulation will be given below.

The total hamiltonian H_t is assumed to consist of three parts:

$$H_t = H_c + H_{sp} + H_{int} \tag{1}$$

where H_c is the hamiltonian of a harmonic vibrator core, H_{sp} is the single-particle shell model hamiltonian and H_{int} is that of the phonon–particle interaction. The chosen basis wavefunctions are those in which $H_c + H_{sp}$ is diagonal and have the form $|\alpha j; NR:IM\rangle$; here α is the set of quantum numbers which completely describe the state of the odd nucleon of angular momentum j; N and R are the number of phonons and the spin of the core state and I is the total angular momentum of the coupled system with M as its z component. Thus, ignoring zero-point energy

$$(H_c + H_{sp})|\alpha j; NR:IM\rangle = (N\hbar\omega + E_j)|\alpha j; NR:IM\rangle \tag{2}$$

where $\hbar\omega$ is the energy of the quadrupole phonon and E_j is that of the odd nucleon.

The particle–phonon interaction, which is taken to be linear in the collective coordinates of the core has the form:

$$H_{int} = \mp k(r)\left(\frac{\hbar\omega}{2C}\right)^{1/2} \sum_{\mu} \{b_\mu + (-1)^\mu b^\dagger_{-\mu}\} Y_{2\mu}(\theta, \phi) \tag{3}$$

which, in the chosen basis, has the matrix elements (Choudhury and O'Dwyer 1967)

$$\begin{aligned}\langle \alpha' j'; N'R':IM|H_{int}|\alpha j; NR:IM\rangle &= \mp(-1)^{I-j'}\left(\frac{\hbar\omega}{2C}\right)^{1/2} \langle \alpha'|k(r)|\alpha\rangle W(RR'jj'; 2I)\langle l'sj' \| Y_2 \| lsj\rangle \\ &\times \{(-1)^{R'}\langle N'R' \| b \| NR\rangle + (-1)^R \langle NR \| b \| N'R'\rangle\}\end{aligned} \tag{4}$$

where C is the nuclear surface deformation parameter and $W(RR'jj'; 2I)$ is a Racah coefficient. The expression for the reduced matrix elements of the single-particle operator $Y_{2\mu}(\theta, \phi)$ has been given by de-Shalit and Talmi (1963); the choice of sign in (3) and (4) which originates from this matrix element is negative for a particle and positive for a hole. The reduced matrix elements of the phonon annihilation operator $b_{2\mu}$ are a factor of $(2R'+1)^{1/2}$ larger than those tabulated by Choudhury (1954) and differ by a factor $(-1)^{R+R'}$ from the values tabulated by Raz (1959)†. The selection rules for the interaction matrix element (4) are: $\Delta N = 1$, $\Delta R \leqslant 2$, $\Delta j \leqslant 2$ and $\Delta l = 0, 2$.

† For a clarification of the subject of these phases see Heyde and Brussaard (1967, p 84).

The matrix elements $\langle \alpha' j' | k(r) | \alpha j \rangle$ are a measure of the coupling strength. We made the usual assumption that they are independent of α and α' except for a phase factor given by Covello and Sartoris (1967). Their magnitude was treated as a parameter k. This is usually expressed in terms of a dimensionless parameter ξ defined by

$$\langle \alpha' j' | k(r) | \alpha j \rangle = (-1)^{n'-n} k = (-1)^{n'-n} (2\pi\hbar\omega C/5)^{1/2} \xi \tag{5}$$

where n is the principal quantum number for the single-particle state αj.

From the diagonalization of H_t the energies and wavefunctions of the coupled system were obtained and the latter were expressed in terms of their expansion coefficients

$$|E; IM\rangle = \sum_{\substack{\alpha \\ N}} \sum_{\substack{j \\ R}} a_E(\alpha j; NR:I) |\alpha j; NR:IM\rangle. \tag{6}$$

The coefficients $a_E(\alpha j; NR:I)$ were then used in the calculation of E2 and M1 matrix elements.

Detailed expressions for the E2 and M1 transition rates and static moments can be found elsewhere in the literature (for example, Heyde and Brussaard 1967, Choudhury and O'Dwyer 1967); they are given here in the following abbreviated form:

$$B(\mathrm{E2}; I \to I') = (2I'+1)\left\{\frac{3}{5}R_0^2\left(e_p - \frac{Ze}{A^2}\right)A_1 + \frac{3}{4\pi}ZeR_0^2\left(\frac{\hbar\omega}{2C}\right)^{1/2}B_1\right\}^2 \tag{7}$$

$$Q(E, I) = C(I2I; I0I)(2I+1)^{1/2}\left(\frac{16\pi}{5}\right)^{1/2}\left\{\frac{3}{5}R_0^2\left(e_p - \frac{Ze}{A^2}\right)A_1 + \frac{3}{4\pi}ZeR_0^2\left(\frac{\hbar\omega}{2C}\right)^{1/2}B_1\right\} \tag{8}$$

$$B(M1; I \to I') = \frac{3}{4\pi}\mu_N^2(2I'+1)(g_R D + g_l E + g_s F)^2 \tag{9}$$

$$\mu(E, I) = \mu_N C(I1I; I0I)(2I+1)^{1/2}(g_R D + g_l E + g_s F), \tag{10}$$

where μ_N is the nuclear magneton. The symbols A_1, B_1, D, E, F stand for expressions which involve double summations over the components of initial and final states and hence involve expansion coefficients $a_E(\alpha j; NR:I)$, single-particle or phonon reduced matrix elements and the corresponding geometrical coupling coefficients; the detailed forms are given in the above mentioned references. Equations (7) to (10) involve a number of additional parameters. The quantity e_p is the effective proton charge; R_0 is the nuclear equilibrium radius; g_R and g_s are the effective gyromagnetic ratios for the core and the proton spin, respectively, and g_l is the gyromagnetic ratio for the proton orbital motion.

3. Calculation A: particle–harmonic vibrator coupling

3.1. Calculation details

Since the $1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$ orbitals are nearly full in silver the odd proton in ^{109}Ag was treated as a hole moving in three orbitals and coupled to an $A = 110$ harmonic core of up to three phonons of surface energy. Four unknown parameters were thus involved, the two single-particle relative energies $E_{3/2} - E_{1/2}$ and $E_{5/2} - E_{1/2}$, the phonon energy $\hbar\omega$ and the coupling strength ξ. There are no available data on single-proton

transfer reactions from which the first two parameters can be estimated. Moreover, available theoretical estimates based on pairing calculations do not yield consistent values for the two single-particle relative energies. It was decided therefore to treat both as variable parameters while using the estimates in table 4 as a guide.

A reasonable value for the phonon energy parameter $\hbar\omega$ was assumed to lie between the excitation energy of the lowest 2^+ state in ^{108}Pd (434 keV) and the corresponding energy in ^{110}Cd (658 keV). The coupling parameter ξ was treated as a free parameter.

Diagonalization of spin matrices was carried out for $I^\pi = 1/2^-$ to $I^\pi = 11/2^-$. The four parameters were varied, subject to the above mentioned conditions, until an optimum fit was obtained for the lowest known five negative parity levels in ^{109}Ag. This fit was produced at the following parameter values: $E_{3/2} - E_{1/2} = 0{\cdot}47$ MeV, $E_{5/2} - E_{1/2} = 1{\cdot}01$ MeV, $\hbar\omega = 0{\cdot}532$ MeV and $\xi = 1{\cdot}96$.

3.2. Energy levels and wavefunctions

The calculated energy spectrum is compared with known negative parity experimental levels in figure 1. The dependence of level energies on the coupling parameter is shown in figure 2. The expansion coefficients for the wavefunctions are presented in table 1.

It will be noted from figure 1 that the five fitted levels are well reproduced; in addition, for each of the experimental levels at 1324 keV ($3/2^-$), 1260 keV ($1/2^-$) and 735 keV ($7/2^-$ or $5/2^+$) there is a calculated level of nearly the same energy and the same spin assignment. There are, however, more levels in the theoretical spectrum than those observed, although this may be partly due to the incomplete state of available data on the higher-lying levels.

Although the value of 532 keV obtained for the phonon energy $\hbar\omega$ seems reasonable, the value of the $p_{3/2}$ single-hole energy ($E_{3/2} - E_{1/2} = 470$ keV) is quite low and close to that of $\hbar\omega$. In fact, it is obviously the strong mixing of the single-hole $|3/2; 00:3/2\rangle$ and the one-phonon $|1/2; 12:3/2\rangle$ components into the lowest $3/2^-$ state (table 1) which is mainly responsible for reproducing the observed low energy position of that level. Consequently, the simple core-excitation model's description of the lowest $3/2^-$ and $5/2^-$ levels as members of a one-phonon doublet is lost in the present calculation.

3.3. Electromagnetic transitions and static moments

Quadrupole moment and B(E2) values were calculated using a value of $1{\cdot}2\,A^{1/3}$ fm for the equilibrium nuclear radius R_0. A value of $e_p = e$ was used for the proton effective charge; this was found to give better agreement with experiment than the value of $2e$ which has been used in similar calculations. The parameter $\{(3/4\pi)ZeR_0\}^2(\hbar\omega/2C)$ which is equal to the core $B(\mathrm{E2}; 2^+ \rightarrow 0^+)$ on the vibrational model (Bohr and Mottelson 1953), was chosen such that the correct E2 transition rates from the lowest $3/2^-$ and $5/2^-$ states to the ground state were obtained. The chosen value is $0{\cdot}100\,e^2b^2$ which lies between the measured $B(\mathrm{E2}; 2^+ \rightarrow 0^+)$ value of $0{\cdot}139\,e^2b^2$ for ^{108}Pd and $0{\cdot}087\,e^2b^2$ for ^{110}Cd (Harper *et al* 1971) but closer to the latter. The parameter C is thus fixed at 36 MeV which, combined with the obtained value of ξ yields 10 MeV for the coupling parameter k.

The M1 reduced transition probabilities and static moments were calculated using $g_l = 1$, $g_R = Z/A = 0{\cdot}44$ and $g_s = 4{\cdot}63 = 0{\cdot}83\,g_s$ (free). The value of g_R used agrees, within experimental error, with the measured value of $0{\cdot}34 \pm 0{\cdot}14$ for the first 2^+ state

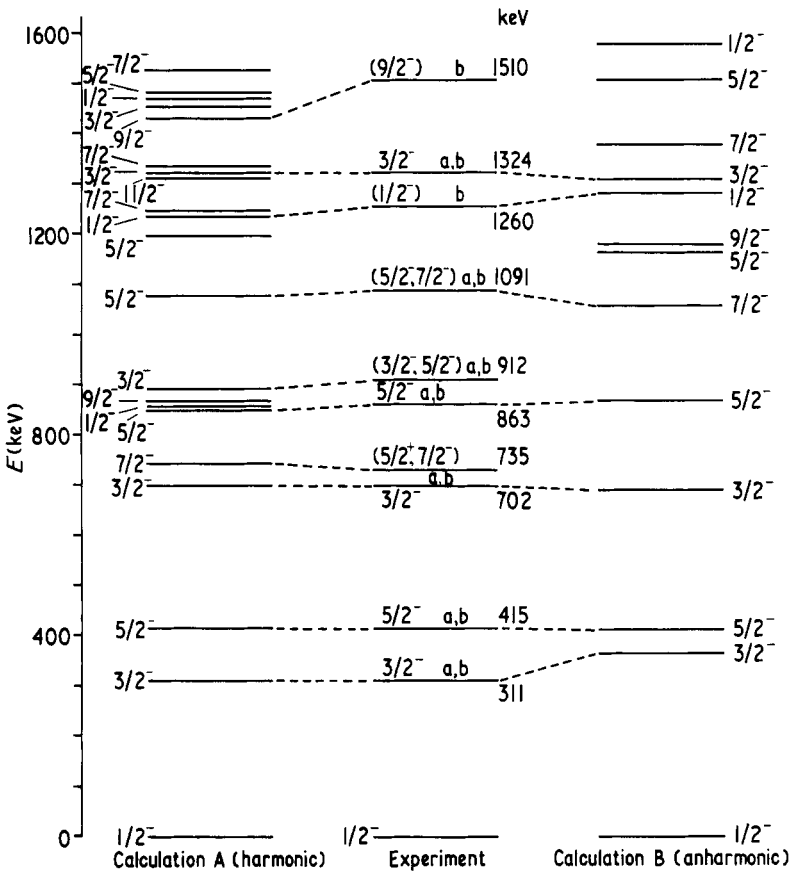

Figure 1. Comparison of calculated and experimental level spectra. Levels marked by (*a*) have been seen in Coulomb excitation work by Robinson *et al* (1970); those marked by (*b*) have been seen in proton inelastic scattering work by Ford *et al* (1970). The 735 keV level was only seen in decay work (Schick and Talbert 1969, Berzins *et al* 1968, Graeffe and Gordon 1967).

in ^{110}Cd (Nuclear Data Sheets 1971a, p 502). The value of the effective spin gyromagnetic ratio g_s was chosen to reproduce the magnetic dipole moment of the ground state.

The calculated E2 and M1 transition probabilities are compared with experiment in table 2; the calculated and experimental electric quadrupole and magnetic dipole moments are given in table 3.

The E2 transitions involving the lowest three states are well reproduced by this calculation (table 2). Further, the general trend of transitions from the second $5/2^-$ (863 keV) level are essentially reproduced. The calculated B(E2) for the ground state transition from the second $3/2^-$ state is the result of some cancellation between the single-hole and the collective contributions but the extent of this is not sufficient to reproduce the very small experimental value.

Although agreement with the experimental B(E2) values is on the whole satisfactory, the calculated values of the quadrupole moments for the lowest $3/2^-$ and $5/2^-$ states are in marked disagreement with experiment in that both values are small in magnitude

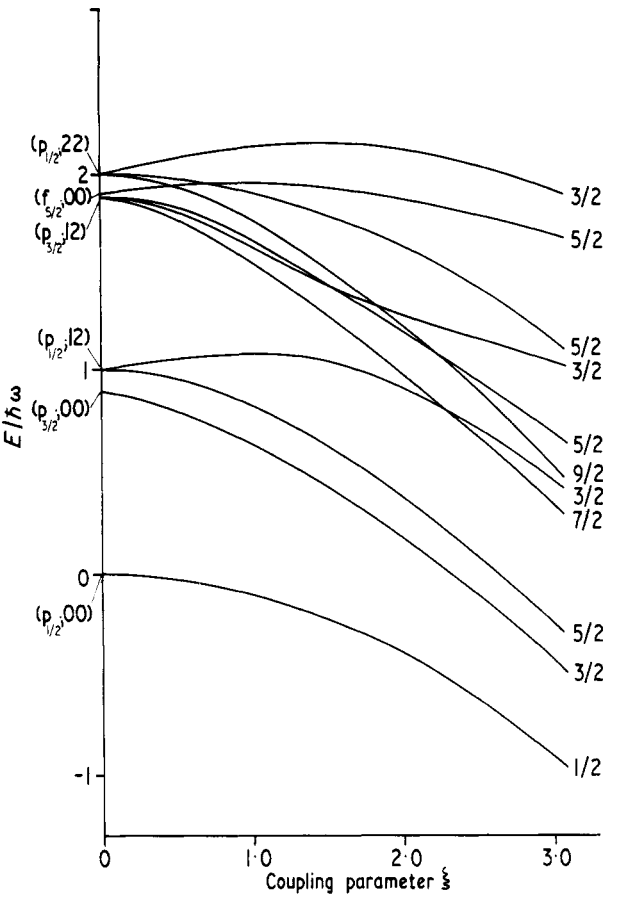

Figure 2. Variation of predicted energy levels (calculation A) with the coupling strength. The other three parameters were fixed at the values obtained from the best fit to the experimental data. The variations of the $1/2^-$ and $5/2^-$ levels arising from the $(p_{3/2}; 22)$ configuration are almost identical and so only the $5/2^-$ level was plotted.

and of the wrong sign. This result points to a basic shortcoming of the calculation and may be traced back to the assumption of a harmonic vibrator as the core of ^{109}Ag. The two even–even neighbours of ^{109}Ag, namely ^{108}Pd and ^{110}Cd, although exhibiting nearly vibrational spectra, both have finite negative quadrupole moments for their lowest 2^+ states (Harper *et al* 1971) in contrast to the harmonic vibrational value of zero. On the other hand, the presently calculated values of quadrupole moments represent the polarizing effect of a proton hole on a harmonic core and hence the positive sign.

The calculated M1 transition probabilities and magnetic dipole moments are in fair agreement with experiment (tables 2 and 3) except for the moment of the lowest $3/2^-$ state and the M1 transitions involving that state. The very small calculated M1 transition rate between the lowest $5/2^-$ and $3/2^-$ states is due to an almost exact cancellation between core and single-hole contributions. These discrepancies probably indicate that the single-hole $|3/2; 00:3/2\rangle$ component in the $3/2^-$ state is much overestimated by the calculation.

Table 1. Coefficients $a_E(\alpha j; NR:I)$ for the states of ^{109}Ag

	$I^\pi = 1/2^-$		$I^\pi = 3/2^-$				$I^\pi = 5/2^-$				$I^\pi = 7/2^-$	
Calculation	A	B	A	B	A	B	A	B	A	B	A	B
E (keV)	0	0	311	364	704	693	418	412	860	871	749	1061
j N R												
1/2, 0 0	0·916	0·896										
3/2, 0 0			0·616	0·446	−0·487	0·527						
1/2, 1 2			0·683	0·834	0·566	−0·338	0·855	0·900	0·244	0·137		
3/2, 1 2	−0·288	−0·276	0·230		−0·423	0·588	−0·102		0·517	0·543	0·633	0·654
3/2, 0 0							0·297	0·272	−0·213	−0·414		
5/2, 1 2	0·240	0·271			−0·140	0·270	0·105		−0·278	−0·387		
1/2, 2 0	0·112											
1/2, 2 2			0·106	0·110	0·386	0·430			−0·673	−0·586		
1/2, 2 4											0·677	0·741
3/2, 2 0			0·121									
3/2, 2 2		−0·141	0·139	0·188	0·204			−0·102	−0·170			
3/2, 2 4							−0·280	−0·218			0·217	0·116
5/2, 2 2		0·134			0·101		0·148	0·166				
5/2, 2 4			0·188	0·209	0·132		0·130	0·128				
3/2, 3 2											0·103	
3/2, 3 3									0·134			
3/2, 3 4									0·134			
5/2, 3 6											0·183	

Amplitudes smaller than 0·1 are omitted. The energies given are the calculated values.

Table 2. Reduced electromagnetic transition probabilities for negative parity states in ^{109}Ag

Transition		E_γ (keV)			$B(E2)\,e^2\,\mathrm{cm}^4 \times 10^{-50}$			$B(\mathrm{M1})\left(\frac{e\hbar}{2Mc}\right)^2 \times 10^{-2}$		
		Calculation			Calculation			Calculation		
I_i	I_f	A	B	Experiment	A	B	Experiment†	A	B	Experiment†
3/2	1/2	311	364	311	10·4	11·1	$11{\cdot}1 \pm 0{\cdot}9$	67·3	37·6	20 ± 6
5/2	1/2	418	412	415	11·0	10·4	$10{\cdot}7 \pm 0{\cdot}9$	0·0	0·0	0·0
	3/2	107	48	104	0·56	0·97	$0{\cdot}8^{+0{\cdot}9}_{-0{\cdot}5}$	0·047	3·5	$4{\cdot}1 \pm 0{\cdot}6$
3/2	1/2	704	693	702	1·96	0·11	$0{\cdot}043 \pm 0{\cdot}010$	22·0	39·7	18^{+12}_{-5}
	3/2	393	349	391	5·61	8·83		43·2	79·2	20^{+14}_{-7}
	5/2	286	281	287	2·48	2·93		5·05	15·4	14 ± 14
5/2	1/2	860	871	863	0·42	0·02	$0{\cdot}58 \pm 0{\cdot}06$	0·0	0·0	0·0
	3/2	549	507	552	4·28	2·13	$1{\cdot}8 \pm 0{\cdot}6$	3·03	13·2	$4{\cdot}9 \pm 0{\cdot}8$
	5/2	442	456	448	11·9	7·32	$2{\cdot}2 \pm 1{\cdot}1$	9·84	17·3	12 ± 2
7/2	3/2	438	697		18·7	12·9		0·0	0·0	0·0
	5/2	331	649		1·88	1·36		77·5	55·7	
1/2	1/2	1241	1286	1260				1·56	0·10	
	3/2	930	922	949	0·34	1·34		43·5	3·79	
	5/2	823	874	845	0·62	1·80		0·0	0·0	0·0
3/2	1/2	1329	1312	1324	0·0006	0·20	$0{\cdot}61 \pm 0{\cdot}09$	0·63	12·1	$<0{\cdot}3$
	3/2	1018	948	1013	0·06	2·77	$0{\cdot}11^{+0{\cdot}09}_{-0{\cdot}07}$	10·1	4·94	10 ± 3
	5/2	911	900	909	0·33	0·85		2·00	0·013	

† The experimental values are taken from Robinson *et al* (1970).

Table 3. Electric quadrupole moments (Q) and magnetic dipole moments (μ) for states in ^{109}Ag

		Q (eb)				μ (nm)		
		Experiment†		Calculation		Experiment‡	Calculation	
E (keV)	I^π	A	B	A	B		A	B
0	$1/2^-$	0	0	0	0	−0·131	−0·131	−0·126
311	$3/2^-$	$-0{\cdot}89 \pm 0{\cdot}11$	$-0{\cdot}63 \pm 0{\cdot}11$	0·27	−0·18	$0{\cdot}87 \pm 0{\cdot}36$	2·34	1·42
415	$5/2^-$	$-0{\cdot}34 \pm 0{\cdot}14$	$-0{\cdot}22 \pm 0{\cdot}14$	0·23	−0·43	$1{\cdot}03 \pm 0{\cdot}48$	0·49	0·87

† Thomas *et al* (1973) give alternative values of the experimental quadrupole moments depending on the (unknown) relative phases of E2 matrix elements. Calculations A and B predict different sets of phases. Hence the experimental quadrupole moments appropriate for comparison with calculation A are different from those for calculation B.
‡ The experimental magnetic dipole moments are from Nuclear Data Sheets (1971b, p 11).

3.4. Discussion

The present harmonic version of the ICM reproduces the low-lying energy levels quite well and, to a lesser extent, some of the E2 and M1 properties. There are, however, notable points of disagreement with experiment, especially in relation to the quadrupole moments of the lowest $3/2^-$ and $5/2^-$ states and the M1 properties involving the lowest $3/2^-$ state.

This suggested that further improvement in the theoretical description of the nucleus may be achieved through one of the following approaches:

(i) Description of ^{109}Ag in terms of three proton holes coupled to a ^{112}Sn core. This method has been applied by Paar (1972) to the positive parity states in ^{109}Ag.

(ii) The method of Bès and Dussel (1969) whereby terms representing third-order anharmonicities are included explicitly in the core hamiltonian. We have attempted the application of this method to the negative parity states in ^{109}Ag but, although quadrupole moments of the proper sign and order of magnitude were obtained, the overall agreement with experimental level energies and electromagnetic transitions was poor.

(iii) The method of Castel *et al* (1971) according to which a quasiparticle is assumed to be coupled to an anharmonic vibrator whose properties are derived from experimental data on neighbouring even–even nuclei. In the limit of weak coupling, and provided the $p_{3/2}$ quasiparticle energy is sufficiently large, this model's description of the lowest three states reduces to that of de-Shalit's core-excitation model. Encouraged by the latter model's success with regard to these states (Throop *et al* 1972) we have applied the above mentioned anharmonic version of the ICM to the negative parity states in ^{109}Ag. The results of this calculation are presented in the next section.

4. Calculation B: quasiparticle–anharmonic vibrator coupling

4.1. Method

The modifications in the formalism of the classical ICM (§ 2) which are entailed by the introduction of anharmonic core vibrations and quasiparticle effects have been discussed in detail by Castel *et al* (1971) and will be presented briefly below:

(i) The core hamiltonian H_c is modified such that the energy of a two-phonon core state of spin R is given by $(2+\eta_R)\hbar\omega$, where the anharmonicity parameters η_R are derived from the experimental core spectrum.

(ii) The assumption that the quadrupole operator Q_μ is proportional to the phonon operator $(b_\mu+(-1)^\mu b^\dagger_{-\mu})$, which underlies the harmonic approximation, is dropped. Thus, instead of using the harmonic values for the off-diagonal and diagonal matrix elements of Q_μ these elements are taken from experimental B(E2) and quadrupole moment values of the core, respectively (equations (7) and (8) in Castel *et al* 1971).

(iii) The single-particle reduced matrix elements which appear in the formulae for the interaction hamiltonian (equation (4)) and for the E2 matrix elements (expression A_1 in equations (7) and (8)) should be multiplied by the factor $(U_iU_j-V_iV_j)$. Where U_j and V_j represent the quasiparticle and quasihole amplitudes respectively in state j. The sign in equation (3) should now be negative, the dependence on whether a particle or a hole is being considered is now taken care of by the quasiparticle factor. A multiplication factor of $(U_iU_j+V_iV_j)$ should be used for the single-particle matrix elements in the formulae for M1 matrix elements (expressions E and F in equations (9) and (10)). A slight departure from the approach of Castel *et al* (1971) was the inclusion of the phase factor $(-1)^{n'-n}$ in the expression for $\langle\alpha'j'|k(r)|\alpha j\rangle$.

4.2. Calculation details

On the anharmonic version of the ICM the negative parity states in ^{109}Ag are described in terms of a quasiproton in the $1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$ orbitals, coupled to the anharmonic vibrations of the ^{110}Cd core†.

Theoretical estimates of the quasiparticle energies E_j and amplitudes U_j were derived (table 4) from two alternative sets of unperturbed single-particle energies ϵ_j.

Table 4. Calculated quasiparticle energies and amplitudes

	Set I			Set II		
	ϵ_j(MeV)	$E_j - E_{1/2}$(MeV)	U_j	ϵ_j(MeV)	$E_j - E_{1/2}$(MeV)	U_j
$f_{5/2}$	−0·085	1·52	0·150	0·61	0·96	0·198
$p_{3/2}$	0·585	0·89	0·199	0·36	1·20	0·177
$p_{1/2}$	1·635	0·0	0·381	1·76	0·0	0·398
$g_{9/2}$	1·950	−0·20	0·492	2·00	−0·15	0·481
Fermi energy λ(MeV)		2·42			2·51	
Energy gap Δ(MeV)		0·78			0·80	

Pairing strength parameter $G = 26/A$ MeV.

Set I was taken from the systematics of Kisslinger and Sorenson (1963) and set II is that given by Bès and Dussel (1969). Although the quasiparticle energies E_j for the two sets differ to the extent of having opposite orderings of the $1f_{5/2}$, $2p_{3/2}$ levels, the calculated quasiparticle amplitudes U_j are quite similar for the two sets and are all of small magnitude. This is an expected result as it implies that the three negative parity orbitals are nearly full in silver and lie well below the Fermi energy. As a result, set I was chosen arbitrarily as the source of U_j values while, as in calculation A, the parameters $E_{3/2} - E_{1/2}$ and $E_{5/2} - E_{1/2}$ were allowed to vary using the estimates in table 4 as a guide. The remaining two parameters, $\hbar\omega$ and ξ, were also treated as in calculation A, that is $\hbar\omega$ was varied between 434 and 658 keV while ξ was treated as a free parameter.

The values used for the anharmonicity parameters were $\eta_0 = 0{\cdot}24$, $\eta_2 = 0{\cdot}24$ and $\eta_4 = 0{\cdot}34$ and were derived from the ^{110}Cd spectrum. Experimental B(E2) values involving one- and two-phonon states in ^{110}Cd (Harper *et al* 1971) were used for the derivation of off-diagonal matrix elements of the Q_μ operator. Two alternative experimental values, $-0{\cdot}55 \pm 0{\cdot}08$ and $-0{\cdot}31 \pm 0{\cdot}07$ *eb*, are given by the same reference for the quadrupole moment of the first 2^+ state in ^{110}Cd; only the former value was used in the final calculation as the use of the latter in preliminary calculations yielded poor agreement with the experimental E2 properties of ^{109}Ag.

As in calculation A, diagonalization was carried out for spin matrices from $I^\pi = 1/2^-$ to $I^\pi = 11/2^-$. The four parameters were varied, subject to the restrictions discussed

† Although treated as a quasiparticle, the proton is thought, on subsequent analysis, to be strongly hole-like. Thus, of the two neighbouring cores, ^{108}Pd and ^{110}Cd, the latter is considered more appropriate and accordingly the matrix elements of the operator Q_μ were derived from experimental data on ^{110}Cd. Fortunately, this choice is not crucial to the calculation since very similar matrix elements are obtained from experimental data on ^{108}Pd (Harper *et al* 1971).

above, until the best fit to the lowest five known negative parity levels in ^{109}Ag was obtained. This was found to occur at the following parameter values: $E_{3/2} - E_{1/2} = 0{\cdot}72$ MeV, $E_{5/2} - E_{1/2} = 1{\cdot}21$ MeV, $\hbar\omega = 0{\cdot}435$ MeV and $\xi = 4{\cdot}5$.

4.3. Energy levels and eigenfunctions

The calculated energy levels are compared with the experimental levels and those from calculation A in figure 1. The dependence of level energies on the coupling strength is shown in figure 3. The expansion coefficients for the wavefunctions are given in table 1. It will be noted from figure 1 that, in addition to the good agreement in energy obtained for the five fitted levels, the experimental levels at 1091 keV ($5/2^-$, $7/2^-$), 1260 keV ($1/2^-$) and 1324 keV ($3/2^-$) are well reproduced. Further, the number of calculated higher-lying levels is about the same as in the experimental spectrum; this comparison could be misleading however since the present calculation included core states of up to two phonons only. Although the quadrupole moment of the core was not treated as a variable parameter, its effect on the energy levels is illustrated in figure 4.

It is interesting to note that the wavefunctions (table 1) of the lowest $3/2^-$ and $5/2^-$ states indicate a predominant single-phonon character. On the other hand the second

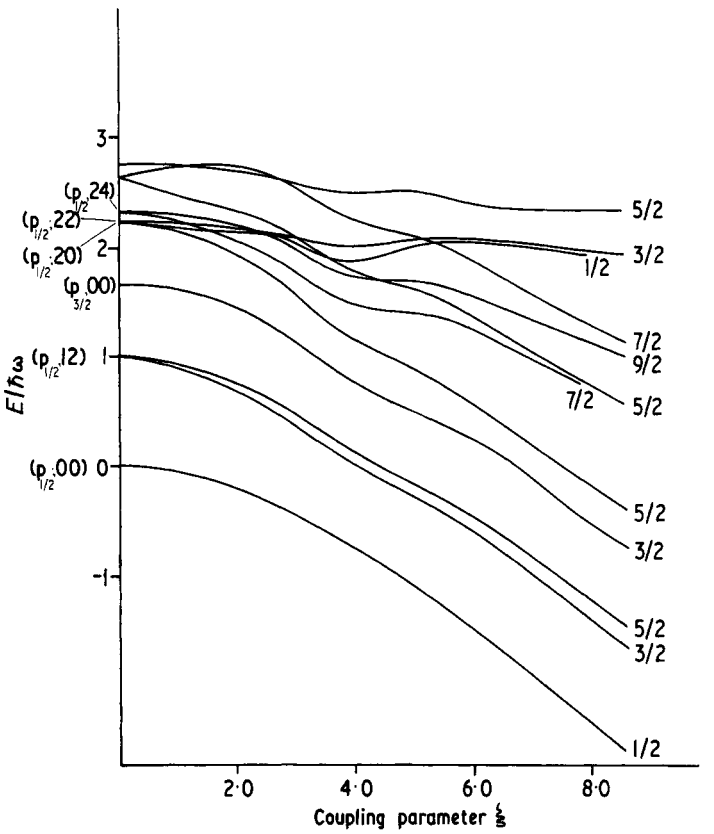

Figure 3. Variation of predicted level energies (calculation B) with coupling strength. The other three parameters were fixed at the values obtained from the best fit to the experimental data.

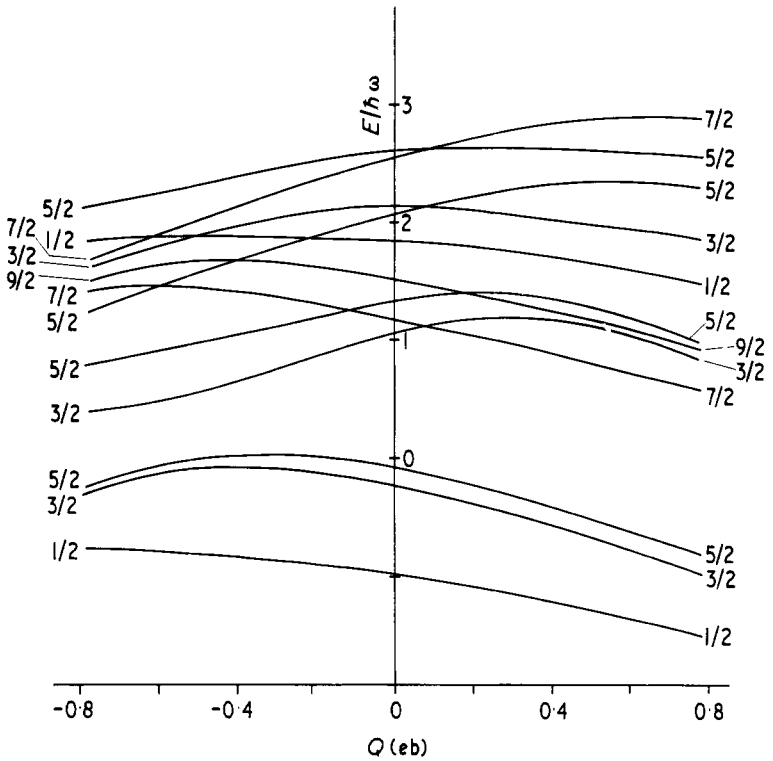

Figure 4. Dependence of predicted level energies (calculation B) on the quadrupole moment of the 2^+ state of the core nucleus.

$3/2^-$ and $5/2^-$ states at 702 and 863 keV, respectively, are only partly two-phonon in character with comparable admixtures of single-phonon and single-particle components. In fact, from coupled-channel analysis of (p, p') data Ford *et al* (1970) estimated the two-phonon components, $|1/2; 22:I\rangle$, in the wavefunctions of the second $3/2^-$ and $5/2^-$ states to have 0·19 and 0·35 probabilities, respectively, in remarkable agreement with the values 0·185 and 0·344 derived from table 1. The same authors estimated that the 1091 keV ($5/2^-$, $7/2^-$) level has a two-phonon $|1/2; 24:7/2\rangle$ component with 0·37 probability which compares reasonably with the value of 0·55 derived from table 1. Ford *et al* (1970) also suggested that the second $1/2^-$ (1260 keV) level has a two-phonon $|1/2; 20:1/2\rangle$ component with 0·15 probability whereas, according to table 1, this is the dominant component in that state.

4.4. E2 and *M1 transition probabilities and static moments*

The calculated E2 and M1 reduced transition probabilities (table 1) and static moments (table 2) were calculated using the modified versions of equations (8), (9), (10), (11) (§ 4.1) and the wavefunctions obtained from the present calculation. The input parameters used were the same as in calculation A.

It will be noted from tables 2 and 3 that good agreement with experiment is obtained for the E2 and M1 moments and transitions involving the lowest five negative parity states and that there is a definite improvement over the rest of calculation A. In particular,

the quadrupole moments for the lowest $3/2^-$ and $5/2^-$ states have now the proper sign and reasonable magnitudes although the moment of the $3/2^-$ state is somewhat underestimated. It is worth noting that in the present anharmonic calculation, as well as in the case of the core-excitation model (Throop *et al* 1972), the values of the quadrupole moments for those two states are essentially determined by the quadrupole moment of the lowest 2^+ core state.

4.5. Discussion

Although the value of 4·5 obtained for the parameter ξ is considerably larger than that of calculation A, the effective coupling strength is not much larger as a result of including quasiparticle effects and using experimental values for the Q_μ matrix elements in calculation B.

The obtained values of $E_{3/2}-E_{1/2} = 0{\cdot}72$ MeV and $E_{5/2}-E_{1/2} = 1{\cdot}21$ MeV for quasiparticle energies are in reasonable agreement with those derived from the systematics of Kisslinger and Sorenson (set I in table 4).

The chosen value of $0{\cdot}100\, e^2b^2$ for the parameter $\{(3/4\pi)ZeR_0\}^2(\hbar\omega/2C)$ is closer to the measured $B(\mathrm{E2}; 2^+ \rightarrow 0^+)$ in ^{110}Cd than to that in ^{108}Pd. On the other hand, the value obtained for the phonon energy, $\hbar\omega = 0{\cdot}435$ MeV, is almost identical to the excitation energy of the 2^+ state in ^{108}Pd rather than being close to that in ^{110}Cd as would be expected. One consequence of this combination of values for the parameters $\hbar\omega$ and $E_{3/2}-E_{1/2}$ is that mixing of the $p_{3/2}$ single-hole component into the lowest $3/2^-$ state is limited. Thus, unlike the case of calculation A, the core excitation model's description of the lowest three states is essentially preserved in the present calculation.

It is worth noting that, although the obtained value of $\hbar\omega$ is lower than expected, the position of the $3/2^-$ state is still about 50 keV too high. In an attempt to obtain a perfect fit to the lowest levels it was found that either the value used for $\hbar\omega$ becomes unreasonably low or the $E_{3/2}-E_{1/2}$ parameter is reduced considerably, leading in both cases to the destruction of the underlying core-excitation picture as well as the overall agreement with experimental E2 and M1 properties. The centre of gravity of the observed $3/2^-$ and $5/2^-$ states is, in fact, lower than the 2^+ excitation energies in both ^{110}Cd and ^{108}Pd. The quadrupole–quadrupole interaction alone is probably not sufficiently adequate to reproduce the low positions and large separation of these levels since it has a limited effect on the lower part of the spectrum where the $p_{1/2}$ orbital is dominating the single-particle subspace. It is possible that the addition of a monopole–monopole term to the interaction hamiltonian would lead to a better energy fit and a larger value of $\hbar\omega$ at the same time.

5. Conclusions

The description of negative parity states in ^{109}Ag in terms of a proton coupled to a harmonic vibrator of quadrupole phonons results in good agreement with experimental level structure and, to a lesser extent, some of the E2 and M1 transition rates. Significant disagreements arise, however, with regard to the quadrupole moments of the lowest $3/2^-$ and $5/2^-$ states and the M1 properties of the $3/2^-$ state.

Considerable improvement in the calculations is achieved through application of the quasiparticle–anharmonic vibrator version of the intermediate coupling model, introduced by Castel *et al* (1971). Good overall agreement with experiment is then

obtained with respect to level energies, E2 and M1 transition rates and electric quadrupole and magnetic dipole moments.

The anharmonic calculation also yields remarkable agreement with coupled-channel analyses of proton inelastic scattering data (Ford *et al* 1970) with regard to the two-phonon content of the second $3/2^-$ and $5/2^-$ states. This agreement also extends to some of the higher-lying states but only qualitatively; in the calculation those levels are expected to be strongly affected by the truncation of core space beyond the two-phonon states.

The present analyses suggest that further improvement in the calculation may be obtained by adding a monopole–monopole term to the interaction hamiltonian and by including more states in the core representation.

Experiments on proton transfer reactions leading to states in ^{109}Ag would obviously provide further useful tests of the present calculation. It would also be of interest to compare the present results with those obtained on the basis of coupling three proton holes to a $Z = 50$ closed shell (Paar 1972).

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